STUDY NOTES

A1 Discount Tuition
PROGRESSIVE EDUCATION

2nd edition

MATHS METHODS
UNIT 3

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MATHS METHODS
2006 VCE REVISION CLASS NOTES

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The notes have been prepared for use as part of the **A1 DISCOUNT TUITION** Unit 3 VCE Revision Class series. Although the notes form an integral part of the Maths Methods revision class, they have been prepared in a way that enables the student to use them on a stand-alone basis.

COORDINATE GEOMETRY & ALGEBRA

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GRAPHS OF POLYNOMIAL FUNCTIONS

GRAPHS OF $y = a(x+b)^2 + B$ **&** $y = \dfrac{A}{(x+b)+B}$

GRAPHS OF INVERSE FUNCTIONS

MODULUS FUNCTION

FACTORISATION OF POLYNOMIALS

ONE-TO-ONE & MANY-TO-ONE FUNCTIONS

GRAPHS OF POLYNOMIAL FUNCTIONS

Linear Functions

- Gradient of a straight line is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Equation of the straight line is $y = mx + c$ or $y - y_1 = m(x - x_1)$
- The angle that a line makes with the positive x-axis can be found by $m = \tan\theta$, $\therefore \theta = \tan^{-1}(m)$
- When 2 lines are parallel, their gradients are the same.
- When 2 lines are perpendicular then the gradient is the negative reciprocal of the gradient of the other. $m_1 m_2 = -1$

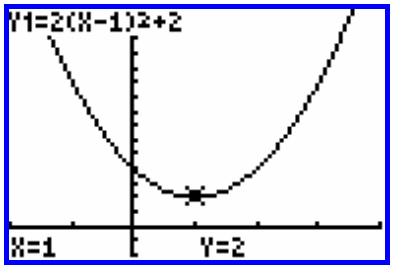
  $m_2 = -\dfrac{1}{m_1}$
- The distance between two points $A(x_1, y_1)$ & $B(x_2, y_2)$ can be found by using Pythagoras Theorem. $d(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The midpoint of a line joining 2 points $A(x_1, y_1)$ & $B(x_2, y_2)$ can be found by $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Quadratic Functions

- The general form of a quadratic equations is $y = ax^2 + bx + c$
- The function can also be given in turning point form $y = a(x - h)^2 + k$
  - Where as the Turning Point is given by $(-h, k)$

Example-1

Draw the graph of $y = 2(x-1)^2 + 2$



The turning point is at (1,2).

To find how many times the graphs crosses the axis, we can use the discriminant, which is given by:

$$\Delta = b^2 - 4ac$$

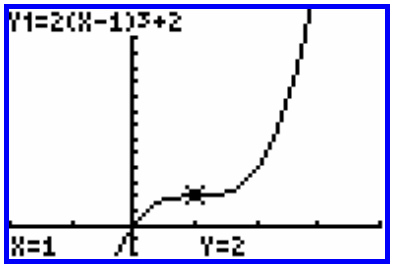
$\Delta < 0$	**No x-axis intercept.**
$\Delta = 0$	**One x-axis intercept.**
$\Delta > 0$	**Two x-axis intercepts.**

Cubic Functions

- The general form of the cubic function is: $y = ax^3 + bx^2 + cx + d$
- Another form is to determine by the intercepts: $y = (x+a)(x+b)(x+c)$
- The other form of cubic function is: $y = a(x-h)^3 + k$
  - Where the point of inflection is given by $(-h, k)$

Example-2

Draw the graph of $y = 2(x-1)^3 + 2$

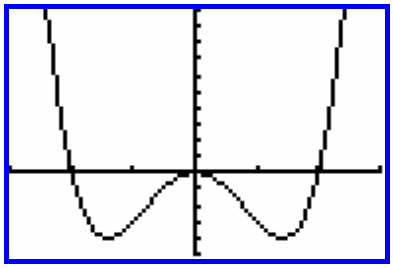


Similarly here, the point of inflection is given by $(-h, k)$, hence $(1,2)$

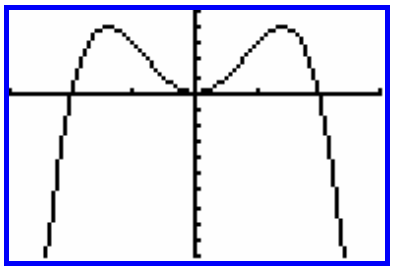
Quartic Functions

- The general form of a quartic is: $y = ax^4 + bx^3 + cx^2 + dx + e$

Positive Shape quartic function looks like this:

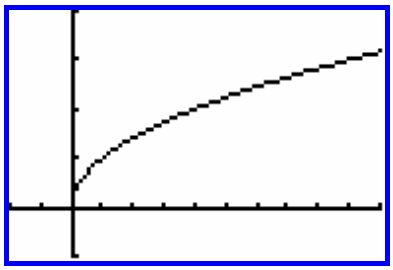


Negative quartic function is upside down:



Square roots

The general form of square root graph is: $y = a\sqrt{x}\,,\ x \geq 0$



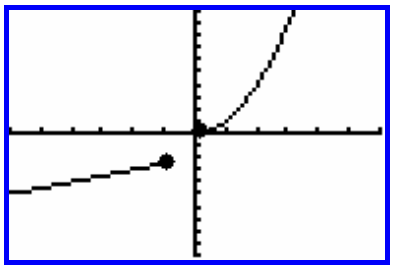
Hybrid Functions

Sketch Hybrid function separately with the given domain.

NOTE: Be careful with the end points. They will vary depending on the domain.

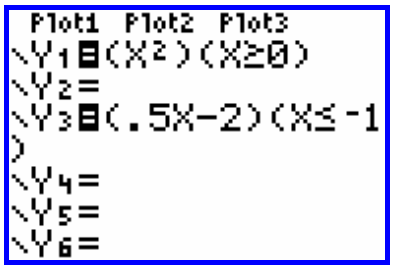
Example-2

Sketch $y = \begin{cases} x^2 & x \geq 0 \\ \frac{1}{2}x - 2 & x \leq -1 \end{cases}$



TI Tricks-1

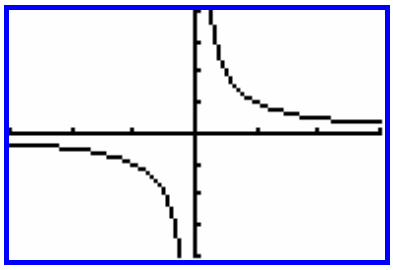
You can type in the limit in TI graphic calculators which will draw the graph according to the limits.



GRAPHS OF $y = a(x+b)^2 + B$ & $y = \dfrac{A}{(x+b)+B}$

Hyperbolae

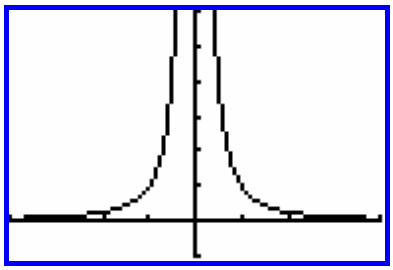
The general form of hyperbolae is: $y = \dfrac{a}{x}, x \neq 0$



- Vertical asymptote at $x = 0$.
- Horizontal asymptote at $y = 0$.

Truncus

The general form of truncus is: $y = \dfrac{1}{x^2}, x \neq 0$



- Vertical asymptote at $x = 0$.
- Horizontal asymptote at $y = 0$.

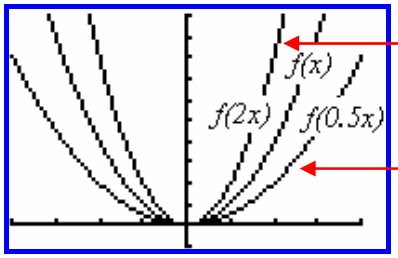
TRANSFORMATION OF GRAPHS

Dilations

$y = 2f(x)$ means $y = f(x)$ has been stretched by a factor of 2 from the x-axis.

$y = f(2x)$ means $y = f(x)$ has been stretched by a factor of "$\dfrac{1}{2}$" from the y-axis.

Explanation



The larger the dilation factor the **THINNER** the graph.

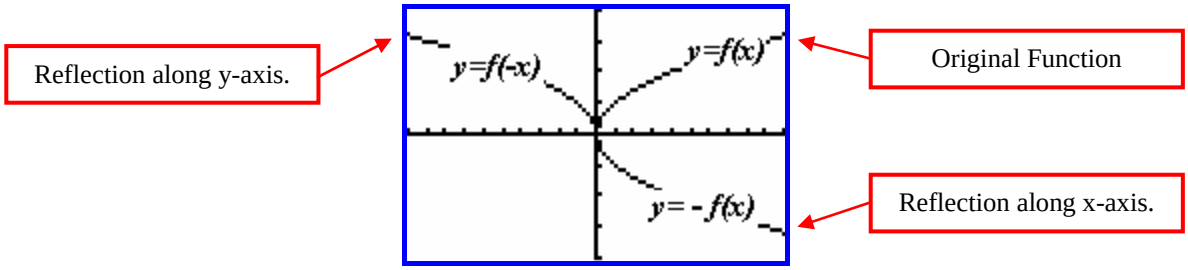
The smaller the dilation factor the **WIDER** the graph.

An example of $y = ax^2$ is shown on the graph above.

Reflections

$y = -f(x)$ means $y = f(x)$ has been reflected (mirrored) along x-axis.
$y = f(-x)$ means $y = f(x)$ has been reflected (mirrored) along y-axis.

Explanation

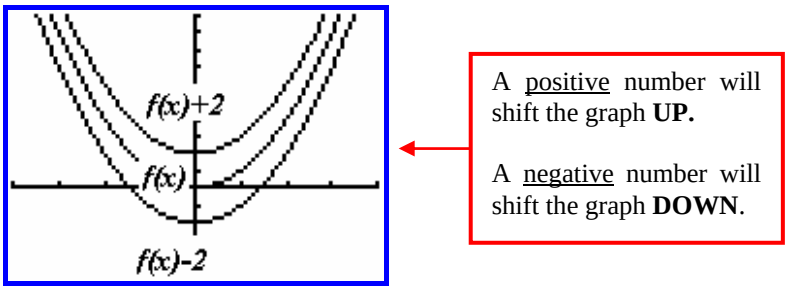


Here we can see that the graph of $y = f(x)$ has been reflected along both x-axis and y-axis.

Translations

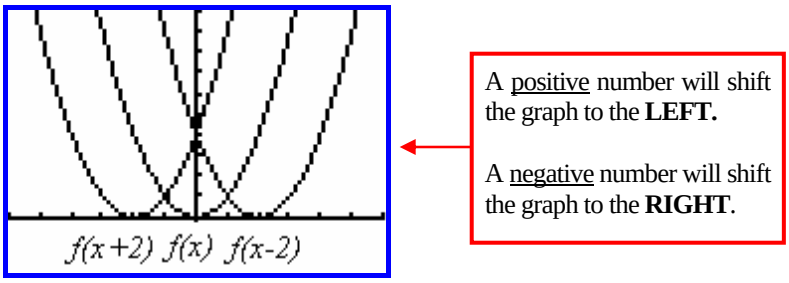
$y = f(x+2)$ means $y = f(x)$ has been translated 2 units to the left.
$y = f(x-2)$ means $y = f(x)$ has been translated 2 units to the right.
$y = f(x)+2$ means $y = f(x)$ has been translated 2 units up.
$y = f(x)-2$ means $y = f(x)$ has been translated 2 units down.

Explanation



Here the graphs are translated upward and downward along the axis.

Explanation



Here the graphs are translated to left and right.

GRAPHS OF INVERSE FUNCTIONS

Inverse Functions

To draw & finding the equation of an inverse function just follow the simple steps described below:

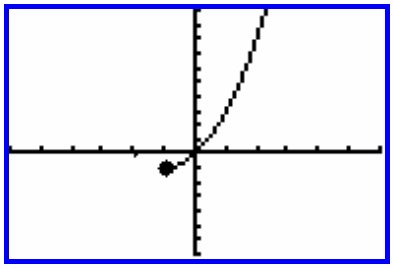
- Write down the domain and range of the original function.
- Interchange x and y
- Make y the subject
- Write the new rules using f^{-1} notation and give the domain and range of f^{-1}.
- Draw the original function.
- Draw the line $y = x$.
- Mirror the original along the $y = x$ plane.

NOTE: If the original function is a one-to-one then the inverse function [f^{-1}] exists. Make sure to check that the original function is a one-to-one function. Do the vertical and horizontal line test to check if it is a one-to-one.

Example-3

For $f:[-1,\infty) \longrightarrow R,\ f(x) = x^2 + 2x$, find the rule of the inverse function and sketch the graph.

- Write down the domain and range of the original function.



The graph of $f(x) = x^2 + 2x$ is drawn above.

Domain: $[-1,\infty)$
Range: $[-1,\infty]$

- Interchange x and y

$y = x^2 + 2x$

$\therefore x = y^2 + 2y$ [Interchange x & y]

$\therefore x = y^2 + 2y + 1 - 1$

$x + 1 = (y+1)^2$

$y + 1 = \pm\sqrt{(x+1)}$

$\therefore y = \pm\sqrt{(x+1)} - 1$

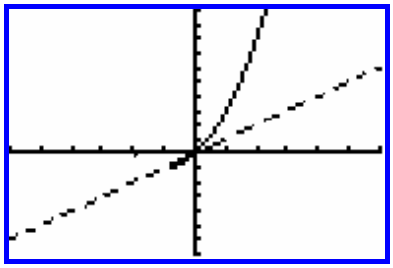
- Write the new rules using f^{-1} notation and give the domain and range of f^{-1}.

The rule of the inverse function is:
$y = \pm\sqrt{(x+1)} - 1$

Domain f^{-1} = Range of f
Range f^{-1} = Domain of f

$\therefore$ Domain $f^{-1} = [-1, \infty)$
$\therefore$ Range $f^{-1} = [-1, \infty)$

- Draw the line $y = x$.



- Mirror the original along the $y = x$ plane.

It is not possible for both of the + & - to be the inverse function.

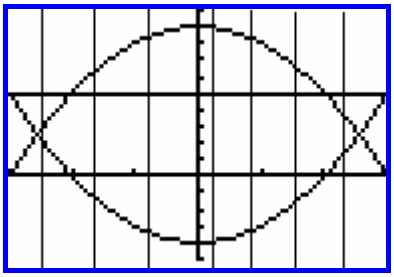
$y = +\sqrt{(x+1)} - 1$ is the inverse function.

Check:

$y = +\sqrt{(-1+1)} - 1 = 0 - 1 = -1$
Where as:
$y = +\sqrt{(1+1)} - 1 = \sqrt{2} - 1 = .4142$. This is not the same as the original function.

For the original function, there is a point at (-1,-1). So for the inverse function it is reversed and giving us (-1,-1) co-ordinate.

Addition of Ordinates



The graph is a constant since $x^2 + (-x^2)$ would leave only the constant.

$y = 5$

Follow the simple steps described below when drawing addition of ordinates.

- Draw the 2 graphs separately first i.e. draw $y = x^2 - 4$ & $y = -x^2 + 9$.
- Simply draw vertical lines along the axis.
- Then look where your vertical line is cutting the graph.
- At that point find the co-ordinate of the original graphs. If you do not know the equation of the graph, then try to estimate.
    - Add up the co-ordinates to get your resultant co-ordinates. (Hence it's called addition of ordinate).

MODULUS FUNCTION

An absolute value is the magnitude of a number disregarding its sign. The absolute value is positive, whether the original expression is positive or negative.

The function $y = |x|$ where $|x|$ means the **absolute value** of the number x.

If $|x|$ denotes the absolute value of a number x, then $|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x < 0$

Example-4

If $|2|$ denotes the absolute value of a number 2, then

$|2| = 2$ for $2 \geq 0$

$|-2| = -(-2) = 2$ for $-2 < 0$

Example-5

Draw the graph of $f(x) = |x-1| + 3$

$f(x) = |x-1| + 3$

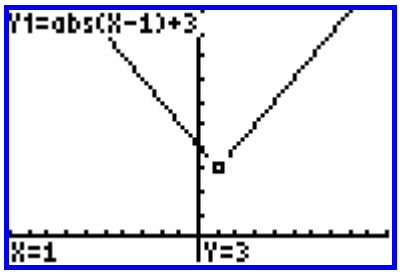
Now $|x-1| = x-1$ when $x \geq 1$

Now $|x-1| = -x-1$ when $x < 1$

$\therefore |x-1| + 3 = x - 1 + 3 = x + 2$ when $x \geq 1$

$\therefore |x-1| + 3 = -(x-1) + 3 = -x + 4$ when $x < 1$

Now we have two equations that we can draw given our domain restriction $x \geq 1$ & $x < 1$. The graph is shown below.



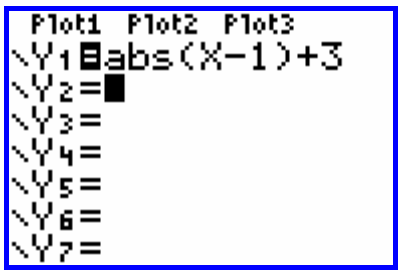
NOTE: Refer to Ti-Tricks to see how to draw this graph on your graphics calculator.

TI Tricks-2

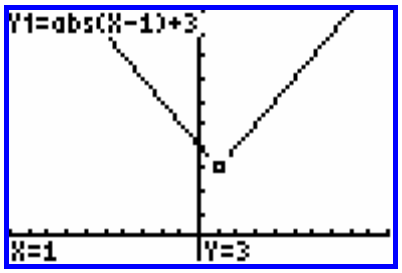
You can draw the Modulus Functions using the "abs(…)" function from the graphics calculator.

To draw a modulus graph, do the following:

1. Go to "Y=".
2. Press "MATH" & go to NUM and choose "abs(".
3. Type the modulus equation as shown below.



4. After you have typed the equation, press the "GRAPH" button to draw the graph as shown below.



FACTORISATION OF POLYNOMIALS

Binomial Theorem

Binomial theorem is used to expand binomial expressions to any power of n.

The general expression is given by:

$$(x+a)^n = \binom{n}{0}x^n a^0 + \binom{n}{1}x^{n-1}a^1 + \binom{n}{2}x^{n-2}a^2 + .. \ldots\ldots\ldots\ldots + \binom{n}{r}x^{n-r}a^r + \ldots.. \binom{n}{n}a^n$$

The coefficients can be calculated by calculating:

$$\binom{n}{0}, \binom{n}{1}, \binom{n}{2} \ldots.. \binom{n}{n}$$

Example-6

$$(x+2)^3 = \binom{3}{0}x^3 2^0 + \binom{3}{1}x^{3-1}2^1 + \binom{3}{2}x^{3-2}2^2 + \binom{3}{3}x^{3-3}2^3$$

$$= x^3 + (3 \times x^2 \times 2) + (3 \times x \times 4) + 8$$

$$= x^3 + 6x^2 + 12x + 8$$

Example-7

Find the coefficient of x^{10} in the expansion of $(x+2)^{20}$.

The $(r+1)$ th term is $\binom{20}{r}x^{20-r}2^r$

When term with x^{10} is, $\binom{20}{10}x^{20-10}2^{10}$

The coefficient is, $\binom{20}{10}2^{10}$

Remainder Theorem

When a polynomial $P(x)$ is divided by $(ax+b)$ the remainder is calculated by evaluating $P\left(-\frac{b}{a}\right)$

Example-8

$P(x) = x^3 + 3x^2 - 5x + 7$ is divided by $2x+1$.

$2x+1=0$

$x = -\frac{1}{2}$

$\therefore x = -\frac{1}{2}$

$\therefore P\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) + 7$

$= -\frac{1}{8} + \frac{3}{4} + \frac{5}{2} + 7 = \frac{81}{8}$

NOTE: If the question does not specifically ask you to use LONG DIVISION then just use Remainder Theorem.

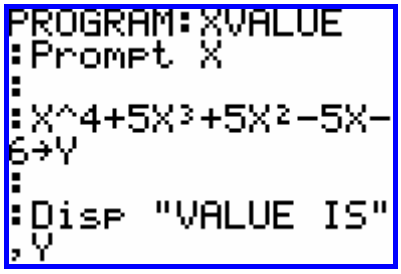
TI Tricks-3

Program: xvalue

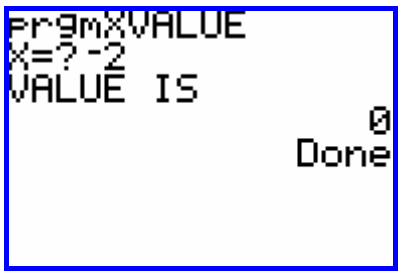
The "xvalue" program is the quickest way to do remainder theorem. The remainder theorem requires you to substitute a value into the equation and find the value. You can use the "xvalue" program; just change the equation (2nd line of the program) to <u>your</u> equation.

To create the program, do the following:

5. Press "PRGM".
6. Go to NEW & press "ENTER".
7. Type the name "XVALUE" in there & press "ENTER".
8. This will take you inside the program, where you have to type in the following:



9. After you have typed everything in, exit & RUN the program. Try any values of x.



So when $f(-2)$ was entered there is no remainder, it's zero.

NOTE: Make sure you change the 2nd line of the "xvalue" program by inserting your own equation, followed by "→ y".

Factor Theorem

If $P(x) = x^3 - 4x^2 - 11x + 30$ is divided by $x - 5$, the remainder is ZERO.
Hence, $x - 5$ is a factor of the $P(x)$.

Now this is another way of doing FACTOR theorem, without having to do LONG DIVISION.

TIP: This is very quick and easy way, so practice this method few times.

STEP 1:
Fill up the 3rd Line with you FACTOR.

$P(x) = x^3 - 4x^2 - 11x + 30$
$=$
$= ....(x-5)....(x-5)....(x-5)$

STEP 2:
Start multiplying numbers into the 3rd line to match the second line.

$P(x) = x^3 - 4x^2 - 11x + 30$
$= x^3 - 5x^2$
$= x^2(x-5)....(x-5)....(x-5)$

STEP 3:
We have to multiply the first factor by x^2 in order to match the first polynomial x^3.

$P(x) = x^3 - 4x^2 - 11x + 30$
$= x^3 - 5x^2$
$= x^2(x-5)....(x-5)....(x-5)$

STEP 4:
Now we have $-5x^2$, but the given value is $-4x^2$. So we must add $+1x^2$ to make the equation balanced.

$P(x) = x^3 - 4x^2 - 11x + 30$
$= x^3 - 5x^2 + 1x^2 - 5x$
$= x^2(x-5) + x(x-5)....(x-5)$

Now, to get that $+1x^2$ in the second line, means that we have to multiply by x to the second factor.

Thus, it gives us -5x after multiplying it in.

STEP 5:

Now we have $-5x$, but the given value is $-11x$. So we must add $-6x$ to make the equation balanced.

$$P(x) = x^3 - 4x^2 - 11x + 30$$
$$= x^3 - 5x^2 + 1x^2 - 5x - 6x + 30$$
$$= x^2(x-5) + x(x-5) - 6(x-5)$$

Now, to get that $-6x$ in the second line, means that we have to multiply by -6 to the third factor.

Thus, it gives us +30 after multiplying it in.

THE EQUATION IS BALANCED.

STEP 6:

Now just collect the similar terms. And take the common factor out.

$$P(x) = x^3 - 4x^2 - 11x + 30$$
$$= x^3 - 5x^2 + 1x^2 - 5x - 6x + 30$$
$$= x^2(x-5) + x(x-5) - 6(x-5)$$
$$= (x-5)(x^2 + x - 6)$$

STEP 7:

Simplify $(x^2 + x - 6)$ further.

$$P(x) = x^3 - 4x^2 - 11x + 30$$
$$= x^3 - 5x^2 + 1x^2 - 5x - 6x + 30$$
$$= x^2(x-5) + x(x-5) - 6(x-5)$$
$$= (x-5)(x^2 + x - 6)$$
$$= (x-5)(x^2 + 3x - 2x - 6)$$
$$= (x-5)[x(x+3) - 2(x+3)]$$
$$= (x-5)(x+3)(x-2)$$

And Long division is done in ONLY 6 lines of writing.

This is an extremely time saving method. So practice this method until you fully comprehend it.

ONE-TO-ONE & MANY-TO-ONE FUNCTION

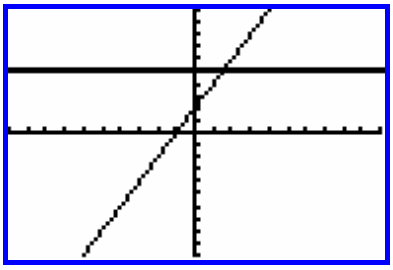
In order to check whether it's a function, we use the "vertical line test". To check what kind of function it is we will use the "horizontal line test".

Below are a few different types of graphs using the "horizontal line test" in order to check what type of functions they are.

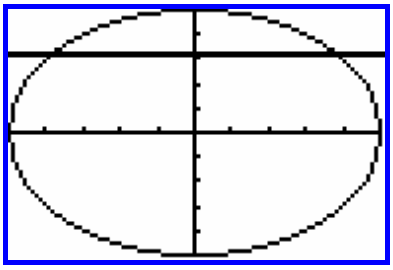
Explanation

If the "horizontal line" crosses only once, then it will be a "One-to-One" function. If it crosses more than once then it will be a "Many-to-One" function.

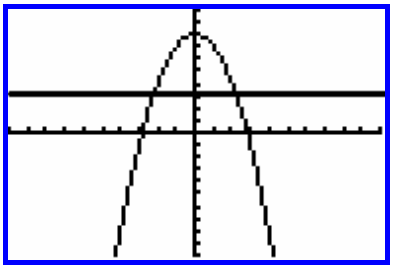
ONE TO ONE



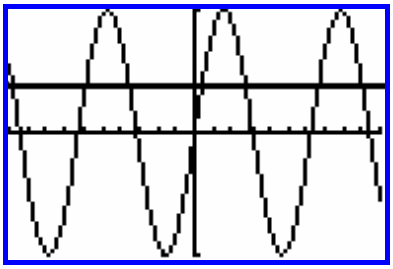
MANY TO ONE



MANY TO ONE



MANY TO ONE

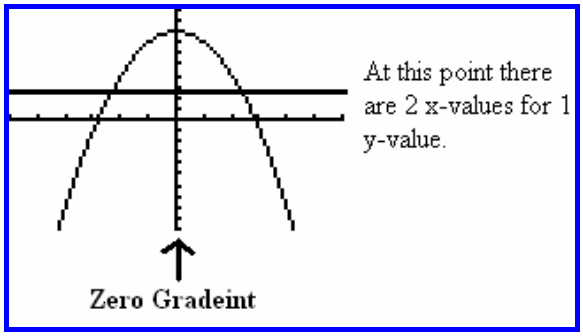


Domain Restriction

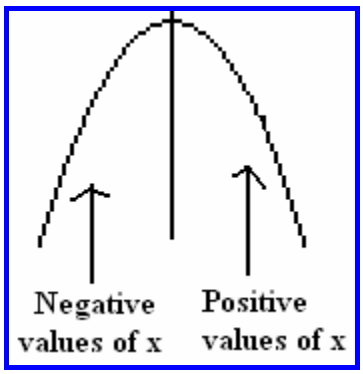
The domain can sometimes be restricted in order to change the type of function.

Example-9

Inverse function exists for what values of x?



Here the turning point is at $x = 0$



So, we can restrict the domain $x \leq 0$ or $x \geq 0$, by doing so it will become one-to-one function.

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SATURDAY MAY 27th 2006

ACCOUNTING	**2:00pm – 5:00pm**

SATURDAY JULY 8th 2006

FURTHER MATHS	**10:30am – 1:30pm**
MATHS METHODS	**2:00pm – 5:00pm**

SUNDAY JULY 9th 2006

ENGLISH	**10:30am – 1:30pm**
ECONOMICS	**2:00pm – 5:00pm**
LEGAL STUDIES	**2:00pm – 5:00pm**

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Rifat has been current with the VCE syllabus throughout its changes since 1997. Completing his VCE with an outstanding score of 95.15 implies that he is well equipped in understanding what the requirements of the various VCE subjects are. Having more than 6 years of private tutoring experience and assisting most of his many students on a regular, long term basis, ensures that he can also pass on the required information in a manner that is understandable and easy for the student to digest.

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