STUDY NOTES

A1 Tuition
PROGRESSIVE EDUCATION

1st edition

MATHS METHODS
UNIT 1 & 2

AMY PAUL

MATHS METHODS
UNIT 1 & 2 STUDY NOTES

$24.95

- PRINTED IN FULL COLOUR.
- FORMULAS & EQUATIONS.
- DIAGRAMS & TABLES.
- GUIDELINES FOR SCIENTIFIC WRITING.
- STUDY METHODS & EXAM STRATEGY.
- 100% AUSTRALIAN PRODUCT.

WARNING: Keep away from water, moisture and dampness, otherwise ink may run or smudge.

CONTENTS

INTRODUCTION

Hello and welcome to Year 11. In this course we are going to be going back through the mathematics that you learnt in years 9 and 10 and expand their definitions a little more. This study guide summarises the whole course, but also gives an overview in different places. These overviews allow you to use a number of different units at the same time, as well as answering a simple question.

For this reason you may find this guide a little jumbled. The ideas I present in this guide are I assure you in some form of order. However, some subject often repeats themselves, so I have merged them together to form little summary at the end.

Throughout this course I want you to create your own summaries for the end of year exams. When you come to a test you should summarised the chapter in your book. For mid year and end of year exams, you should revise these topics and merge your summaries together. Most schools expect you to have 2 A4 pages (both sides) summaries.

The final piece of advice I am going to give is that Year 11 is the stepping stone to Year 12. Year 11 may seem less stressful as it is not really counted as part of your final VCE marks. This may be the case however in Year 12 your topics from Year 11 are covered. If you do not pay attention in Year 11 and seek help when you are really in trouble it could cause serious problems in Year 12!

The algebra section in Unit 1 may be attempted in Unit 2. The syllabus is actually the same for both units. Better than repeating myself, I have just left the algebra section as a whole in Unit 1.

I wish you the best of luck with the course and hope that you find this guide very useful. Please refer to the study and exam hints at the end of the guide as they will give you further clues into the course.

Amy Paul

FUNCTIONS & GRAPHS

FUNCTIONS & GRAPHS

COORDINATE GEOMETRY

FUNCTION NOTATION

LINEAR GRAPHS

QUADRATIC GRAPHS

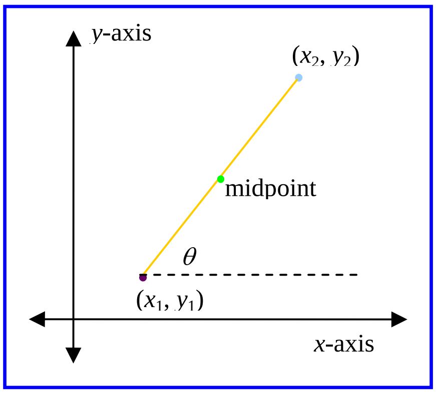
CUBIC GRAPHS

QUARTIC GRAPHS

OTHER GRAPHS

COORDINATE GEOMETRY

Gradient of a straight line



- Gradient is denoted by the letter *m* in equations.
- Gradient is the slope of a line
- $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$
  - where (x_1, y_1) and (x_2, y_2) are the two points either end of the line and θ is the angle of the slope as displayed in the picture.

Formulas for finding the equation of a straight line

There are three formulas here that are handy to find the equation of a straight line. However they are all interrelated as I will show shortly. Whenever you find the equation of a straight line, your final answer must be displayed as the general formula for a straight line which is $ax + by + c = 0$, where a, b and c are all constants. For more information please see the section of sketching a straight line.

Last year you should have come across our first equation;

$$\boldsymbol{y = mx + c}$$

where m was the gradient and c was y-intercept. This equation is still highly useful in finding the straight line of an equation if you know the gradient and the y-intercept. Given any point on the line you could easily find c by replacing x and y for that point and solving for c which is also a good way to find the equation of a straight line.

The equation we just looked at sort of devalues the need for our next equation. Given any point on a line, (x_1, y_1), and the gradient m we get:

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

This equation will work every time for any point and gradient. However the first equation can also do that function as well.

Our last equation is a hybrid of the second equation. Often when looking for the equation of a straight line, we actually don't know the gradient. We can always just find the gradient first and then plug it into one of the above equations, however, that is a waste of time when we can just substitute the gradient formula for m giving us:

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

This equation eliminates the need for anything else than just any two points on the line.

Distance Formula

The distance between any two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular lines

- A line that is perpendicular to another line means that it is at right angles or 90^o is the angle between the two lines.
- Perpendicular to is often depicted by the symbol ⊥
- The equation of a perpendicular line to the gradient is often called the normal
- If two lines are perpendicular, then it is said that the products of their gradients equal -1.

$$m_1 \times m_2 = -1$$

Midpoint of a segment

The midpoint, *M*, of a segment can be found by this formula:

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Example: Consider the two points (3, 3) and (9, 2)

a Find the distance of the line:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(9-3)^2 + (2-3)^2} = \sqrt{36+1} = \sqrt{37}$$

b Find the equation of that line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3 = \frac{2-3}{9-3}(x-3)$$

$$6y - 18 = -x + 3$$

$$x + 6y - 21 = 0$$

c Find the equation of the perpendicular line that goes through the midpoint of that line.
Finding Gradient of line:

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{9-3} = \frac{-1}{6}$$

Gradient of perpendicular line:
$m_1 m_2 = -1 \Rightarrow m_2 = 6$

Finding midpoint:

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{3+2}{2}, \frac{9+3}{2}\right) = \left(\frac{5}{2}, 6\right)$$

Equation of line:

$$y - y_1 = m(x - x_1)$$

$$y - 6 = 6\left(x - \frac{5}{2}\right)$$

$$y - 6 = 6x - 15$$

$$6x - y + 9 = 0$$

FUNCTION NOTATION

- Brackets are used all the time is function notation. Below I have outlined what each set of brackets means
  - {…} refers to a normal set of elements.
  - [*a*, *b*] refers to a closed ordered range of elements; $a \le x \le b$
  - (*a*, *b*) refers to an opened ordered range of elements; $a < x < b$
  - [*a*, *b*) is a mixture of the two; $a \le x < b$
  - (*a*, *b*] is a mixture of the two; $a < x \le b$

- Set notation:
  - $\notin$ - is not an element of
  - $\in$ - is an element of
  - $\subset$ - is a subset of
  - $\not\subset$ - is not a subset of (or not contained in)
  - $\cup$ - union with
  - $\cap$ - intersection with
  - $\varnothing$ - null set, or empty set
  - \ - excluding
  - {(*a*, *b*), (*c*, *d*), …} is a set of ordered pairs
  - A relation is a set of ordered pairs.

- Special sets
  - ***N*** refers to the set of natural numbers (0, 1, 2, 3, 4, …)
  - ***J*** refers to the set of integers (…-2, -1, 0, 1, 2, …)
  - ***Q*** refers to the set of rational numbers.
  - ***Q'*** refers to the set of irrational numbers.
  - ***R*** refers to the set of real numbers
  - $\boldsymbol{R} \subset \boldsymbol{Q} \subset \boldsymbol{J} \subset \boldsymbol{N}$

- Relations and graphs
  - The independent variable (domain) is shown on the horizontal axis
    - Domain of a relation is the set of first elements of a set of ordered pairs (*x* axis)
  - The dependent variable is shown on the vertical axis of a graph
    - Range of a relation is the set of second elements of a set of ordered pairs (*y* axis)
  - Domain and range is described by brackets [,], (,).

- $f(x) = \ldots$ is used to describe a function of *x*. If you want to evaluate $x = 2$, you need to find $f(2)$.

- Functions are described by the domain and the rule.

  $f: X \to Y,\ f(x) = \ldots$
  - *X*: Domain
  - *Y*: Co-domain
  - …: Rule
- Dom *f* is the abbreviation of the domain of $f(x)$

- Ran *f* is the abbreviation of the range of $f(x)$

LINEAR GRAPHS

The general formula for a straight line graph is

$$ax + by + c = 0$$

This is the standard that you are expected to leave all linear graphs, when you are asked to find them algebraically. If we rearrange this graph in to an equation we were introduced to in Year 10, called the gradient intercept form we get:

$$y = \frac{-a}{b}x - \frac{c}{b}$$

Giving us a general formula for the finding the gradient and the *y*-intercept:

Gradient: $m = \frac{-a}{b}$ *y*-intercept: $c = \frac{-c}{b}$

If we solve for the *x*- intercepts we get:

$$x\text{-intercept} = \frac{-c}{a}$$

There are two ways to graph linear graphs;

- using the gradient intercept method
  - This method involves plotting the *y*-intercept and then marking in the next point using the gradient, drawing a line through both points.
- plotting the *x* and *y* intercepts
  - This method involves finding both *x* and *y* intercepts and drawing a line through both points.
  - This method is handy as in VCE you are expected to mark the values of the *x* and *y* intercepts.
  - This method sometimes is not handy, as our *x* and *y* intercepts are the same through the origin, or we have the equation $x = a$ or $y = a$.

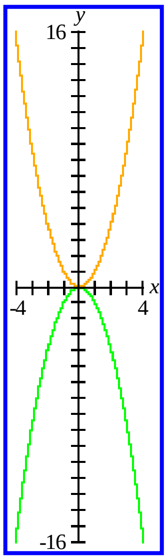
Graphing the equations $x = a$ and $y = a$;

- $x = a$: this is a straight vertically line that is parallel to the *y*-axis, and therefore never goes through it. This line hits that *x*-axis at $x = a$
- $y = a$: this is a straight horizontal line that is parallel to the *x*-axis, and therefore never goes through it. This line hits that *y*-axis at $y = a$

QUADRATIC GRAPHS

The general formula for a quadratic graph is:

$$y = ax^2 + bx + c$$



The graph in general is a curved line called a parabola, which is either concaved up or down, and has one turning point, as can be seen in the basic graph to the right. In orange the graph of $y = x^2$ and in green $y = - x^2$. As the graph get more complicated the turning points move the position, and the graph could grow thicker or skinnier.

The best way to sketch a quadratic graph is by finding the three intercepts, which is outlined in the method below:

1. Find the x-intercepts, by solving $y = 0$.
- You can find these using the calculator by entering the equation in $Y =$ and then 2nd → calc → 2: zero.

2. Find the y-intercepts, by solving $x = 0$.

3. Find the turning point can be found by finding the midpoint of the x-intercepts and subbing that x value into the equation to find y. This could also be found using completing the square method. General formula: $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$.

4. Plot all points showing their coordinates.

Another way to easily graph a quadratic graph is via the turning point form, which has the general equation $y = a(x - b)^2 + c$. The beauty of this equation is that it gives the turning point (b, c), while a is the dilation factor (dilation meaning how thick or how skinny the graph can be). The a also gives whether the graph is concave up or down. If it is positive then $a > 0$ then the graph is concave up and if $a < 0$ then the graph is concave down.

The tuning point form can be found by completing the square. (For more information look at the factorising section in the next chapter).

General equation: $y = ax^2 + bx + c$

$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

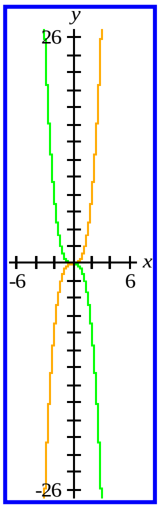
$$y = a\left(x^2 + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

CUBIC GRAPHS

A cubic graph has the general formula:

$$y = ax^3 + bx^2 + cx + d$$

There are two possible graphs that could occur with a cubic. The first graph is the cubic in it's basic for. If we consider the equations $y = x^3$ and $y = -x^3$, which are depicted in the diagram to the left. We can see that the graph generally has a gradient in the same direction either positive or negative, apart from where it passes through the origin, where the gradient is zero. This point is known as a point of inflection, and it is important that you remember this point when sketching this point, as you need to state it with all cubic graphs.



If we take this cubic graph one step further we can move the point of inflection to any place on the *xy* plane, we can also make the graph either skinnier than it already is or fatter. This is done by using the formula

$$y = a(x - b)^3 + c$$

Where (b, c) is where the point of inflection occurs and a is a dilation factor that states how skinny or fat the graph is. If you are expected to graph this type of graph in the exams you will probably be given it in that form.

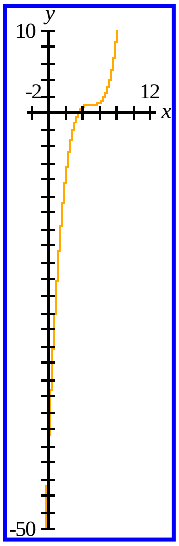
The second type of graph possible from a cubic is a graph with a maximum turning point, a minimum turning point that is two different types of turning points. This graph may already be factorised for you, or in the general equations formula stated at the start. The way you would sketch this graph is via the following method:

1. Determine whether the cubic is either negative or positive. (This can be done by looking at whether a is positive or negative);
2. Find the y intercept (let $x = 0$, or it is most likely the d value);
3. If not already factorised, factorise the expression so we can find the x-intercepts;
4. Find x-intercepts (let factors of $f(x)$ equal 0);
5. Use all available information to sketch the graph;
6. If you have your calculator. Find approximate turning points using at this stage the calculator. (From the end of unit 2 onwards you should be able to use calculus to discover turning points)

Example: graph the following expressions stating domain and range.

a $y = \frac{1}{3}(x - 5)^3 + 1$

You will notice that this graph is an example of the first graph we talked about. Applying the formula we can tell that the turning point for the point of inflection is at point (5, 1). The dilation factor of $\frac{1}{3}$ has stretched our graph in width. This graph hits the x-axis at:



$0 = \frac{1}{3}(x - 5)^3 + 1$

$-3 = (x - 5)^3$

$\sqrt[3]{-3} = x - 5$

$x = 5 + \sqrt[3]{-3} \approx 3.56$ to 2 decimal places

This graph will hit the y-axis at:

$y = \frac{1}{3}(0 - 5)^3 + 1$

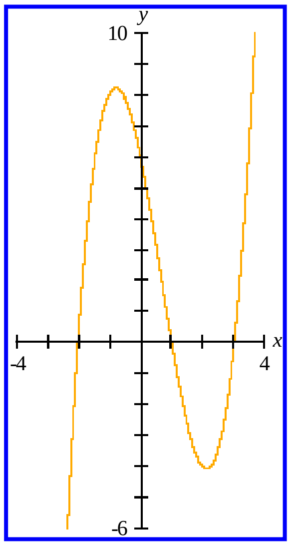
$y = \frac{-122}{3} \approx 40.67$ to 2 decimal places

Domain and range are $x \in \mathbf{R}$ and $y \in \mathbf{R}$ respectively

b $y = x^3 - 2x^2 - 5x + 6$
The graph is positive.
The y-intercept is at $y = 6$
We need to factorise this graph: (for more information on factorising see the algebra section I have done both methods so you can check your own results).
At $x = 1$: $y = 1 - 2 - 5 + 6 = 0 \Rightarrow x - 1$ is a factor

$y = x^3 - 2x^2 - 5x + 6$
$y = x^3 - x^2 - x^2 + x - 6x + 6$
$y = x^2(x - 1) - x(x - 1) - 6(x - 1)$

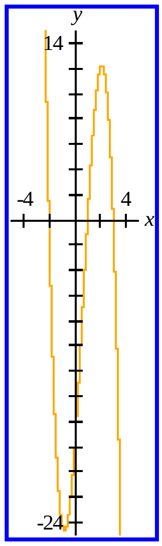
$$\begin{array}{r|llll} & x^2 & -x & +6 & \\ \hline x-1 & x^3 & -2x^2 & -5x & +6 \\ & x^3 & -x^2 & & \\ \hline & & -x^2 & -5x & \\ & & -x^2 & +x & \\ \hline & & & -6x & +6 \\ & & & -6x & +6 \\ \hline & & & & 0 \end{array}$$



$y = (x - 1)(x^2 - x - 6)$
$y = (x - 1)(x + 2)(x - 3) \Rightarrow x$-intercepts at 1, -2, 3
From calculator; the turning points are roughly at: (2.12, -4.06) and (-0.79, 8.21)

Domain and range are $x \in \mathbf{R}$ and $y \in \mathbf{R}$ respectively

c $y = -3x^3 + 6x^2 + 15x - 18$
The graph is negative
y-intercept: $y = -18$



Factorising:
$y = -3(x^3 + 2x^2 + 5x - 6)$
The rest we fully factorised last question: $y = -3(x - 1)(x + 2)(x - 3)$

x-intercepts: $x = 1$, -2, and 3
The -3 acts as a dilation factor. Our graph is about to be skinnier and in the opposite direction to the last question.

From calculator, turning points are at: (2.12, 12.18) and (-0.79, -24.63)

Domain and range are $x \in \mathbf{R}$ and $y \in \mathbf{R}$ respectively

You may have noticed in the last two examples that the graph is practically the same values for the x and larger and opposite for the y. This was done deliberately so you could see the effects first hand and also the difference between positive and negative graphs.

QUARTIC GRAPHS

Quartics are graphs that involve polynomials to the degree of 4. They come in many shapes and sizes and depending on the graph can have one, two, or three stationary points; that it places where the graph is zero gradient.

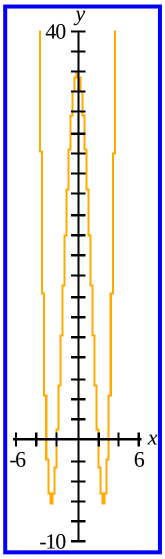
In Mathematical Methods, all you need to worry about are quartics that are already factorised. These will already have been factorised for you, which will make drawing the graph given the information easy.

Factorised general formula:

$$\boldsymbol{y = (x - a)(x - b)(x - c)(x - d)}$$

The method for sketching quartics is simple:

1 Find y intercept: ($f(0) = abcd$)
2 Find x intercepts (let factors = 0, x intercepts at a, b, c, and d)
3 Graph all available information to sketch the graph.
4 If you have your calculator. Approximate the turning points for the graph



Example: sketch $y = (x - 2)(x + 2)(x - 3)(x + 3)$, stating domain and range.
x – intercepts: $x = 2, -2, 3, -3$
y – intercepts: $y = 36$

This graph is symmetrical, so one of the turning points is actually (0, 36) the other two are at approximately: (2.55, -6.25) and (-2.55, -6.25)

Domain: $x \in \mathbf{R}$

Range: $y \in [-6.25, \infty)$

OTHER GRAPHS

Hyperbolae

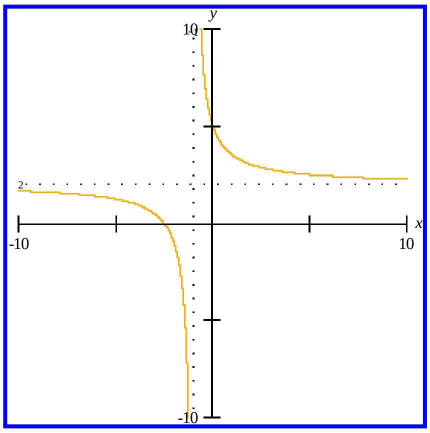
General formula:

$$y = \frac{a}{x - b} + c$$

a is the dilation factor
$x = b$ is the x-asymptote
$y = c$ is the y-asymptote

An asymptote is a line where your graph can never be equal to. The graph will approach it but never touch it.

Example 1: Graph $y = \frac{3}{x+1} + 2$ stating the *x* and *y* intercepts and the domain and range.



Asymptotes at *x* = -1 and *y* = 2.
y-intercepts: at *x* = 0, *y* = 3 + 2 = 5
x-intercepts: at *y* = 0, *x* = $\frac{3+2}{-2}$ ≈ 2.5

dom *f*: **R**\-1

ran *f*: **R**\2

In the graph the asymptotes are dashed in purple while the actual graph is denoted in orange.

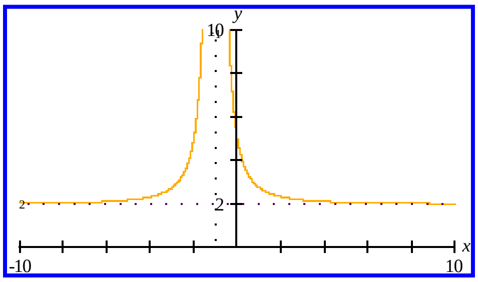
Truncas

$$y = \frac{a}{(x-b)^2} + c$$

a is the dilation factor
Asymptotes at *y* = *c* and *x* = *b*.

Unless *a* is negative, our graph is going to be the positive side of the *y* asymptote

Example 2: Graph $y = \frac{3}{(x+1)^2} + 2$ stating the *x* and *y* intercepts and the domain and range.



There is no *x* intercept as that is the negative part of the graph.
y – intercept: *y* = 5

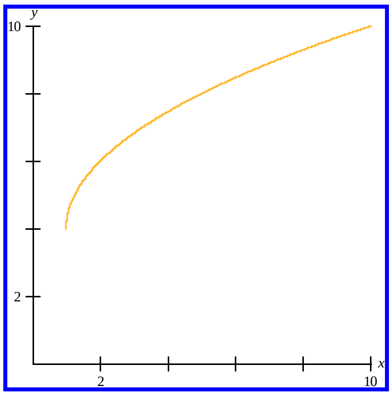
dom *f*: ***R***\-1

ran *f*: (2, ∞)

- $y = c + a\sqrt{x-b}$

This graph is half a parabola. If you rearranged the equation to make *x* subject you will see that we end up with a y^2 which means a parabola which is laying on its side. As in our equation to start with our *x* values do not have a ± sign in front of it, we only take the positive side of this parabola, unless *a* is negative.
In the graph *a* is the dilation factor, it also determines which half we graph.
(*b*, *c*) is the point where our graph starts.

Example 3: Graph $y = 4 + 2\sqrt{x-1}$ stating domain and range.



There is no *x* or *y* intercepts in this case.

Dom *f*: $[1, \infty)$

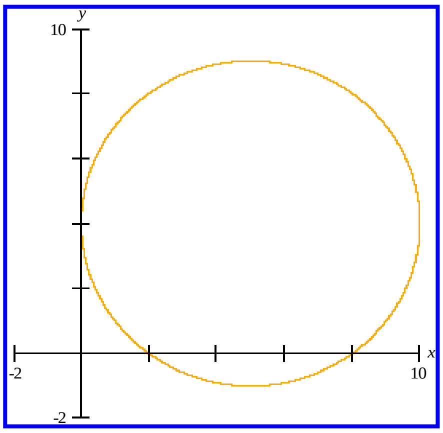
Ran *f*: $[4, \infty)$

Circles

$$(x - h)^2 + (y - k)^2 = r^2$$

This graph graphs a circle with radius *r* and centre at (*h*, *k*)

Example 4: Graph $(x - 5)^2 + (y - 4)^2 = 25$



Centre: (5, 4) marked in green dashed line

Radius 5 units which means circle goes through points (0, 4), (5, 9), (10, 4) and (5, -1) on the dashed lines $x = 5$ and $y = 4$.

x – intercepts: (2, 0) and (8, 0)
y – intercepts: (0, 4)

dom *f*: [0, 10]

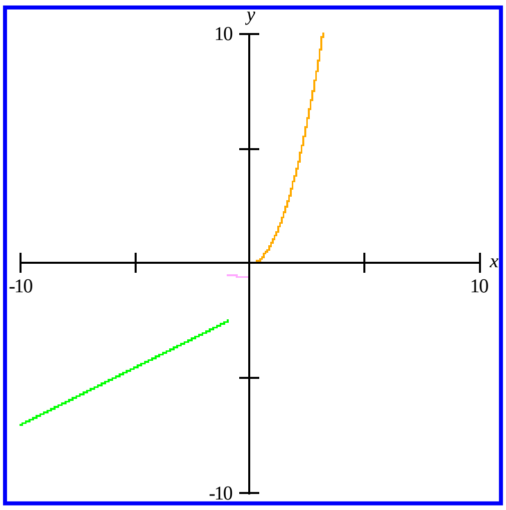
ran *f*: [-1, 9]

Hybrid Functions

Hybrid functions are a mixture of different functions. That is they have a different rule for different sections of the domain. You sketch them according to the domains given in the graphs.

Example 5: Sketch the following graph stating domain and range.

$$y = \begin{cases} x^2 & x \geq 0 \\ \dfrac{2}{x+5} - 1 & -1 < x < 0 \\ \dfrac{1}{2}x - 2 & x \leq -1 \end{cases}$$



Finding bounds on graphs:
$y = x^2$; starts at (0, 0) and continues on to ∞
$y = \dfrac{2}{x+5} - 1$; begins at $\left(-1, -\dfrac{1}{2}\right)$, but doesn't touch the point and ends at $\left(0, -\dfrac{3}{5}\right)$ but never touches the point.

$y = \frac{1}{2}x - 2$; begins ends at $\left(-1, -2\frac{1}{2}\right)$ it keeps going to -∞.

Dom *f*: **R**

Ran *f*: **R**\ $\left\{-2\frac{1}{2} < y < -\frac{3}{5}, -\frac{1}{2} < y < 0\right\}$

PLEASE NOTE: I have graphed each part in a different colour deliberately so you can see how it comes together. The graph would normally be sketched in the same colour.

GRAPHING SUMMARY

Below are steps to follow for any graph. Throughout this chapter we have seen many different types of functions and how they are graphed individually. However, it can be tedious to remember every step of every function, so I have summarised this to a few general steps below:

1 Find all boundaries, intercepts and asymptotes.
2 Plot these values and hopefully connect the dots
3 If you are unsure where the graph is meant to go with all this information, sub in some points to get an idea.

GRAPHICAL MODELLING

- Linear modelling
  - Total cost = fixed cost + Cost per unit × Number of units.

- Quadratic and cubic modelling
  - You are expected to solve problems modelled on quadratic and cubic functions. Some values you will need to find using a calculator or a computer graphing package such as mathcad or excel. One common example is the turning point on cubic and quartic functions.

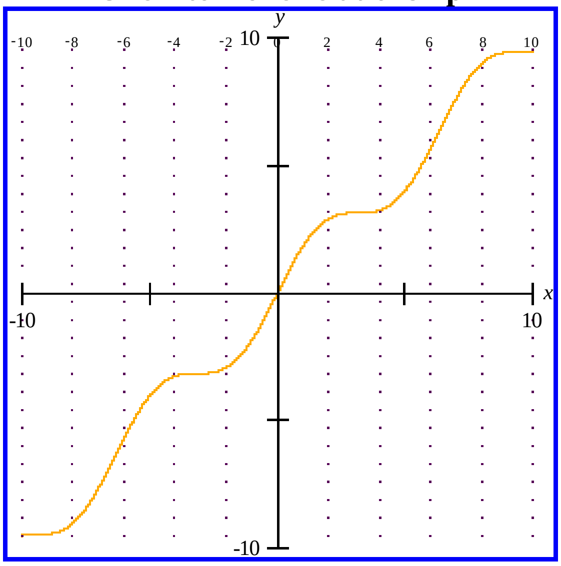
VERTICAL TEST FOR A FUNCTION

A function must have a one-to-one ratio. That is for every *x*-value there is at most one *y*-value and vice versa. A way to for a function is the vertical line test. The way it works is vertical lines (parallel to the *y* axis) are drawn through the graph. If the graph is a graph of a function, there will be only one *x* value that each line hits.

With graphs there are several possibilities for different graphs:

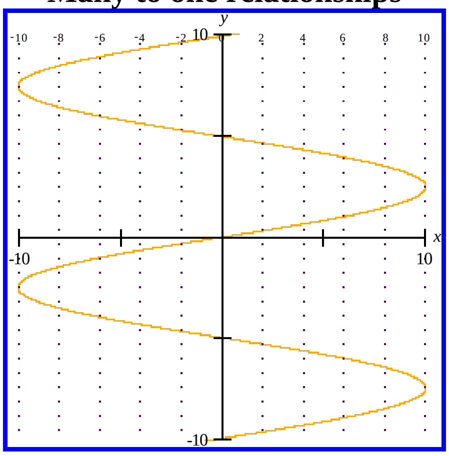
FUNCTIONS

One – to – one relationship

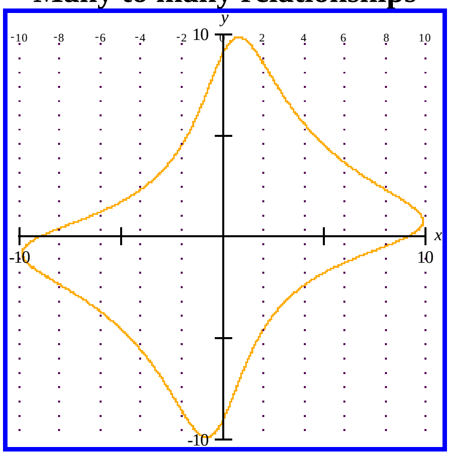


NOT FUNCTIONS

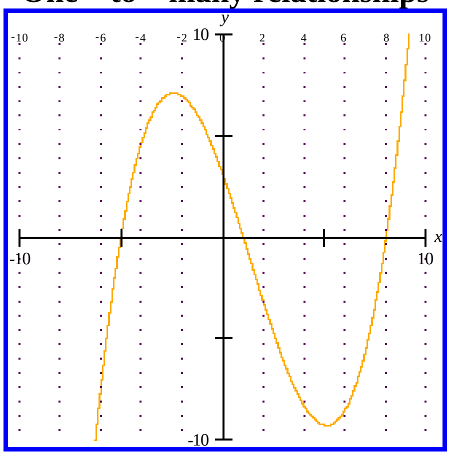
Many to one relationships



Many to many relationships



One – to – many relationships



ADVERTISEMENT

VCE ADVANTAGE CLASSES

This is probably our most powerful service in terms of providing students with an advantage over their peers. Our **ADVANTAGE CLASSES** are aimed at teaching students within 15 hours what they are required to learn during the whole semester. Students who attend at least 7 classes will receive a FREE Unit 3 VCE Revision Class ticket for the subject. If started early, students will actually learn the subject before other students do. Fun and interactive sessions.

- ***Learn the VCE syllabus in 15 hours – use school time for revision while others are learning.***
- ***10 sessions of 1½ hour duration each.***
- ***Complementary full colour study notes provided for each class.***
- ***Small groups – we need a minimum of 4 students to attend.***
- ***Flexible timetable – we start the first session once a minimum of 4 students can make it on any given time and day.***
- ***Location: Holmesglen TAFE in Chadstone – ample free parking and right next to bus stop and train station.***
- ***Cost: $40 per session, or $350 prepaid.***
- ***Check website for details and to register your interest.***

SUBJECTS COVERED:

Accounting
Chemistry
Economics
Further Maths
Maths Methods
Specialist Maths

Visit the “VCE Advantage Classes” section of our website to reserve your place.
www.a1tuition.com.au

STUDY NOTES

First edition published 2007.
Latest print runs in October 2007.

A1 TUITION
P.O. BOX 1188
CAULFIELD NTH 3161
VICTORIA, AUSTRALIA

Designed in Australia.
Bound in Australia.
Printed in Australia.

AUTHOR

AMY PAUL
Bachelor of Science
Bachelor of Teaching (Primary and Secondary) – Completing final year

Amy has been current with the VCE system throughout its changes since 2000. She is experienced, up to date with the current curriculum and this experience has enabled her to identify key problem areas that students face when dealing with the subject at the VCE level.
Amy attained her first paid tutoring job in 2000. In 2003 Amy also gained a position as an author for Heinemann Education, writing 4 mathematics CD-ROMs, to help students and teachers with their schoolwork. In 2005 Amy began her formal training as a teacher.

One of Amy's ethos is not only to help the students with the subject she is paid to teach, but also to help them to formulate good study routines, for every subject they study. She also has a strong ethos about having up to date information and strives to find more resources to help her students in their studies.

Amy offers full service private and group tuition with comprehensive study notes in the following VCE subjects:

Chemistry
Maths Methods
Specialist Maths

TUITION FEES

	1 STUDENT	2 STUDENTS	3 STUDENTS	4 STUDENTS
STANDARD FEE	$75/1½ hrs	$112.50/1½ hrs	$131.25/1½ hrs	$140.00/1½ hrs
FEE PER STUDENT	$75/1½ hrs	$56.25/1½ hrs	$43.75/1½ hrs	$35.00/1½ hrs

CONTACT AMY

(03) 9455 0895